Confining Configurations in QCD and Relation to Rigid Strings

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The gauge field configurations of QCD gauge fields in the infrared regime are obtained by magnetic symmetry condition. The effective dual action exhibits dual Meissner effect with quarks included. A string representation of this action corresponds to rigid string.

1. Introduction

A description of confinement in QCD requires the presence of magnetic monopoles[1]. The resulting effective theory can be interpretted as producing dual Meissner effect [2] responsible for confinement. Lattice QCD studies indicate that monopoles cover the physical vacuum in the confining phase[3]. Confinement is in the infrared regime where the QCD coupling is large so that one has to use non-perturbative analysis. In the non-confining phase, in the ultra-violet regime, we realize the important property, 'asymptotic freedom', by using the full $SU(3)$ gauge field configurations $A^a_\mu(x)$. In the confining phase, we have the monopole dominance and a partial (subset of) $A^a_\mu(x)$ will be sufficient to realize confinement through dual Meissner effect. This will be best described by the 'magnetic symmetry' [4], which reveals the topological nature (of monopoles) of QCD. This symmetry restricts the dynamical degrees of freedom while keeping the gauge degrees of freedom.

2. Confining gauge field configurations

The 'gauge covariant magnetic symmetry' condition for $SU(3)$ is

$$D^{ab}_\mu \omega^b \equiv \partial_\mu \omega^a + g f^{acb} A^c_\mu \omega^b \ = \ 0, \qquad (1)$$

where $\omega^a(x) \in SU(3)$ and are chosen as

$$\sum_{a=1}^{8} \omega^a\omega^a = 1; \sum_{b,c=1}^{8} d^{abc}\omega^b\omega^c = \frac{1}{\sqrt{3}}\omega^a, \qquad (2)$$

d^{abc} and f^{abc} are the symmetric and anti-symmetric tensors of $SU(3)$ respectively. These two conditions on the octet vector ω^a are $SU(3)$ invariant. A general solution to (1) is obtained as

$$A^a_\mu(x) = C_\mu(x)\omega^a - \frac{4}{3g} f^{abc}\omega^b(\partial_\mu\omega^c), \qquad (3)$$

where $C_\mu(x)$ is an arbitrary vector field, independent of ω^a. The above gauge field configuration will be taken here to describe QCD in the confining phase and will be shown to produce dual Meissner effect and as well as the confining rigid string action. By letting $\omega^a\lambda^a \ = \ \Omega$ (λ^a's are the Gell-Mann matrices), with the conditions (2), the eigenvalues of Ω are $-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$. There are two distinct eigenvalues and therefore [5] the magnetic symmetry is governed by the little group $U(2)$ of Ω. The field strength associated with (3) is found to be

$$F^a_{\mu\nu} = (\partial_\mu C_\nu - \partial_\nu C_\mu)\omega^a - \frac{4}{3g} f^{abc}(\partial_\mu\omega^b)(\partial_\nu\omega^c), (4)$$

and is "$SU(3)$ parallel" to ω^a, that is, $f^{abc}F^b_{\mu\nu}\omega^c \ = \ 0$ consistent with the identity $[D_\mu, D_\nu]^{ab}\omega^b = if^{abc}F^b_{\mu\nu}\omega^c$, in view of (1). However, this does not imply that $F^a_{\mu\nu}$ is along ω^a.

3. Effective Dual Action

Using the above field strength and the gauge field configuration (3), the standard QCD action with the quarks becomes

$$S = -\frac{1}{4}\int\{f^2_{\mu\nu} - \frac{8}{3g} f_{\mu\nu}X_{\mu\nu} + O(\frac{1}{g^2})\}d^4x$$

$$+\int\{\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi-\frac{g}{2}\bar{\psi}\gamma^{\mu}\lambda^{a}C_{\mu}\omega^{a}\psi\}d^{4}x$$
$$+\int\{\frac{2}{3}\bar{\psi}\gamma^{\mu}\lambda^{a}\psi f^{abc}\omega^{b}(\partial_{\mu}\omega^{c})\}d^{4}x, \qquad (5)$$

where we have denoted $\partial_\mu C_\nu - \partial_\nu C_\mu$, the Abelian part of $F^a_{\mu\nu}$ by $f_{\mu\nu}$ and $f^{abc}\omega^a(\partial_\mu\omega^b)(\partial_\nu\omega^c)$, the topological part of $F^a_{\mu\nu}$ by $X_{\mu\nu}$. It then follows $\partial_\mu X_{\mu\nu} \neq 0$, $\partial_\mu \tilde{X}_{\mu\nu} \neq 0$, where $\tilde{X}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}X_{\alpha\beta}$. Further,

$$-\frac{2}{3}\oint \epsilon_{\mu\nu\alpha\beta}f^{abc}\omega^{a}(\partial_{\alpha}\omega^{b})(\partial_{\beta}\omega^{c})dx^{\mu}\wedge dx^{\nu}, \qquad (6)$$

is a topological invariant [6] and therefore $X_{\mu\nu}$ can be taken to describe monopoles.

Introducing $\mathcal{G}_{\mu\nu}$ as dual to $f_{\mu\nu}$ in (4), the variation with respect to the C_μ-field yields an equation for $\mathcal{G}_{\mu\nu}$, which is solved to obtain

$$\mathcal{G}_{\mu\nu}=\epsilon_{\mu\nu\lambda\sigma}\partial_{\lambda}\tilde{A}_{\sigma}+\frac{4}{3}X_{\mu\nu}+\frac{g}{2\pi^{2}}\int d^{4}y\frac{1}{\mid x-y\mid^{4}}$$
$$\{(x-y)_{\mu}J_{\nu}(y)-(x-y)_{\nu}J_{\mu}(y)\}, \qquad (7)$$

where $\tilde{A}_\sigma$ is dual to C_μ and the current $J_\mu = \frac{1}{2}\bar{\psi}\gamma^\mu\lambda^a\psi\omega^a$. It also follows from the variational equation that the current J_μ is conserved, $\partial_\mu J_\mu = 0$. Using this, we obtain the dual action

$$S_{d}=\int[-\frac{1}{4}\tilde{f}^{2}_{\mu\nu}-\frac{4}{3}\tilde{A}_{\sigma}\partial_{\lambda}\tilde{X}_{\lambda\sigma}+\frac{4}{9}\tilde{X}_{\mu\nu}\tilde{X}_{\mu\nu}$$
$$-\frac{g}{4\pi^{2}}\tilde{A}_{\mu}\tilde{J}_{\mu}+\frac{g}{4\pi^{2}}\tilde{X}_{\mu\nu}\tilde{Y}_{\mu\nu}+\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi$$
$$+\frac{g^{2}}{8\pi^{2}}\int d^{4}yJ_{\mu}(x)(x-y)^{-2}J_{\mu}(y)$$
$$-\frac{4}{3}j^{a}_{\mu}f^{abc}\omega^{a}\partial_{\mu}\omega^{c}]d^{4}x, \qquad (8)$$

where $\tilde{f}_{\mu\nu} = \partial_\mu\tilde{A}_\nu - \partial_\nu\tilde{A}_\mu$, $\tilde{J}_\sigma = \epsilon_{\mu\nu\lambda\sigma}\partial_\lambda Y_{\mu\nu}$, the dual current, where $Y_{\mu\nu} = \int \frac{d^4y}{|x-y|^4}\{(x-y)_\mu J_\nu(y) - (x-y)_\nu J_\mu(y)\}$ and $j^a_\mu = \bar{\psi}\gamma^\mu\lambda^a\psi$. In here, we have the dual Abelian field $\tilde{A}_\sigma$ coupled to the dual current $\tilde{J}_\sigma$ and a Biot-Savart energy term for the quarks. All the fields are massless and the dual action (6) has dual $U(1)$ gauge invariance.

4. Generation of mass for dual Abelian field

In order to realize dual Meissner effect, the dual gauge field must acquire mass. Instead of introducing a scalar field with its own potential, we generate a mass term by quantum fluctuations and using 'gauge mixing mechanism' [7]. We first consider the quantum fluctuations of $\tilde{A}_\mu$ and $\tilde{X}_{\mu\nu}$ and integrate over the fluctuations of $\tilde{A}_\mu$. We do not encounter Gribov problem as these fields are Abelian. The integration of the fluctuations of the dual Abelian field produces the following additional terms,

$$\frac{4}{9}\tilde{x}^{2}_{\mu\nu}+a(\partial_{\mu}\tilde{x}_{\mu\nu})^{2}-\frac{4}{3}\tilde{A}_{\sigma}(\partial_{\lambda}\tilde{x}_{\lambda\sigma}), \qquad (9)$$

where $\tilde{x}_{\mu\nu}$ is the fluctuation of $\tilde{X}_{\mu\nu}$ and $'a'$ is a dimensionful parameter coming from the integration of the fluctuations of $\tilde{A}_\mu$. We now have a massive anti-symmetric tensor field $\tilde{x}_{\mu\nu}$ coupled to massless dual field $\tilde{A}_\sigma$. We still have dual $U(1)$ gauge invariance. Representing $\partial_\lambda\tilde{x}_{\lambda\sigma}$ by $\partial_\sigma\xi$, the equation of motion $\partial_\sigma\xi = \frac{2}{3a}\tilde{A}_\sigma$, is used to obtain the action

$$S_{d}=\int[-\frac{1}{4}\tilde{f}^{2}_{\mu\nu}-\frac{4}{3}\tilde{A}_{\sigma}\partial_{\lambda}\tilde{X}_{\lambda\sigma}+\frac{4}{9}\tilde{X}^{2}_{\mu\nu}-\frac{g^{2}}{4\pi^{2}}\tilde{A}_{\sigma}\tilde{J}_{\sigma}$$
$$+\frac{g}{4\pi^{2}}\tilde{X}_{\mu\nu}\tilde{Y}_{\mu\nu}-\frac{4}{9a}\tilde{A}^{2}_{\sigma}\ +\ Diracterms\ ]d^{4}x, \qquad (10)$$

thereby generating mass term for the dual Abelian field.

5. Dual Meissner effect

The equation of motion for $\tilde{A}_\sigma$ from (8) is

$$\partial_{\mu}\tilde{f}_{\mu\nu}=\frac{4}{3}\partial_{\mu}\tilde{X}_{\mu\nu}+\frac{g^{2}}{4\pi^{2}}\tilde{J}_{\nu}+m^{2}\tilde{A}_{\nu}, \qquad (11)$$

where $m^2 = \frac{8}{9a}$. It follows from the above expression that $\partial_\mu\tilde{A}_\mu = 0$. The above expression is used to eliminate $\tilde{A}_\mu$ from (8) to finally obtain an effective action in the infrared regime as

$$S_{eff.}=\int[-\frac{8}{9}\partial_{\lambda}\tilde{X}_{\lambda\nu}(\partial^{2}-m^{2})^{-1}\partial_{\rho}\tilde{X}_{\rho\nu}+\frac{4}{9}\tilde{X}^{2}_{\mu\nu}$$
$$+\frac{g^{4}}{32\pi^{4}}\tilde{J}_{\nu}(\partial^{2}-m^{2})^{-1}\tilde{J}_{\nu}-\frac{g^{2}}{3\pi^{2}}\tilde{J}_{\nu}(\partial^{2}-m^{2})^{-1}\partial_{\lambda}\tilde{X}_{\lambda\nu}$$
$$+\frac{g}{4\pi^{2}}\tilde{X}_{\mu\nu}\tilde{Y}_{\mu\nu}+\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi$$
$$+\frac{g^{2}}{8\pi^{2}}J_{\mu}(x)\int d^{4}y(x-y)^{-2}J_{\mu}(y)$$
$$-\frac{4}{3}j^{a\mu}f^{abc}\omega^{a}(\partial_{\mu}\omega^{c})]d^{4}x. \qquad (12)$$

The first term is as in the London theory of Meissner effect which confines the monopole configurations and the second term corresponds to mass for these objects. The third term confines quarks through $\tilde{J}_\mu$ by Dual Meissner mechanism. In addition we have a coupling of quark current and monopole configuration. The dual field strength $\tilde{X}_{\mu\nu}$, which contains the monopoles, can be interpretted as Abrikosov flux lines responsible for the confinement of quarks and they themselves are confined.

The structure of the quark current J_μ or its dual, has $\lambda^a \omega^a = \Omega$. In a representation in which Ω is diagonal, this endows magnetic charges to the coloured quarks. There are two magnetic charges corresponding to the two distinct eigenvalues of Ω. This allows the construction of mesons and baryons with zero magnetic charge and the magnetic charge neutrality of the physical hadrons ensures the colour neutrality.

6. A string representation

To obtain a string representation of the above infrared effective dual action, we consider the situation without quarks. Quarks can be included directly to the resulting string action, by coupling them to the worldsheet and this just renormalizes the string couplings without introducing new terms [8]. The monopole configuration $X_{\mu\nu}$ is represented now by [9]

$$X_{\mu\nu} = \int d^2\xi \; \delta^4(x-y) \; [y_\mu, y_\nu], \tag{13}$$

where $y_\mu(\xi^1, \xi^2)$ represents a point on the worldsheet swept by the string of the monopole, $[y_\mu, y_\nu] = \epsilon^{ab}\partial_a y_\mu \; \partial_b y_\nu$, with $\partial_a = \frac{\partial}{\partial \xi^a}$ and it can be seen that $[x_\mu, x_\nu]^2 = 2g$ where g is the determinant of the induced metric (first fundamental form) on the worldsheet, namely, $g_{ab} = \partial_a x^\mu \partial_b x^\mu$. Then using the above representation for $X_{\mu\nu}$, we obtain the string action

$$\begin{aligned} S_{string} \;=\; & \frac{m^2}{9\pi} K_0(\frac{m}{2\Lambda}) \int \sqrt{g} d^2\xi \\ & - \frac{\Lambda^2}{18\pi m^2} \int \sqrt{g} g^{ab} \partial_a t_{\mu\nu} \; \partial_b t_{\mu\nu} \; d^2\xi \\ & ,+ \frac{\Lambda^2}{m^2} \int \sqrt{g} R \; d^2\xi \end{aligned} \tag{14}$$

where Λ is an ultra-violet cut-off, K_0 is the modified Bessel function and $t_{\mu\nu} = \frac{1}{\sqrt{g}}[x_\mu, x_\nu]$. The first term is the Nambu-Goto area term in the string action, the second term is the Polyakov-Kleinert extrinsic curvature (second fundamental form) action [10] and the third one is the Euler characteristic of the string world sheet. Thus the string representation of the effective dual action derived here corresponds to rigid string.

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